The Study of Complex Manipulation via Kinematic Hand Synergies: The Effects of Data Pre-Processing

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***Abstract*— The study of kinematic hand synergies through matrix decomposition techniques, such as singular value decomposition, supports the theory that humans might control a subspace of predefined motions during manipulation tasks. These subspaces are often referred to as synergies. However, different data pre-processing methods lead to quantitatively different conclusions about these synergies. In this work, we shed light on the role of data pre-processing on the study of hand synergies by analyzing both numerical simulation and real kinematic data from a complex manipulation task, i.e., piano playing. The results obtained suggest that centering the data, by removing the mean, appears to be the most appropriate pre-processing technique for studying kinematic hand synergies.**

***Index Terms*— Synergy, Singular Value Decomposition, Human Hand, Manipulation, Task Complexity, Rehabilitation**

I. INTRODUCTION

The human hand is the end-effector that serves humans in interaction with themselves, the external environment, and other individuals [1]. It is an anatomically complex system comprised of 27 bones and 30 muscles [2] that are controlled to perform highly dexterous manipulation tasks [3], [4], [5]. All in all, the hand presents more than 20 degrees-of-freedom (DOFs) [2], thus providing a highly redundant control space to interact with the environment which, for a rigid object, has at most 6 DOFs.

An average of 800,000 individuals in the US suffer from Cerebral Vascular Accident (CVA) [6] every year, while roughly 700,000 upper-limb hand amputees currently live in the United States [7], [8]. Losing hand functionality poses a great barrier for these individuals in a society designed for humans with dexterous hands. This has a major detrimental impact on their quality of life as they lose their capability to perform numerous activities of daily living. To improve the available rehabilitation and assistive technologies for impaired individuals (e.g., hand prostheses and rehabilitative exoskeletons), it appears necessary to deepen the understanding of how humans control and move their hands.

Among the many research questions about the human hand, there is growing interest in comprehending how humans seamlessly move a highly redundant system without particular apparent effort. Santello et al. were the first to suggest that kinematic synergies of the hand (i.e., linear combinations of joint motions) could provide a reasonable reduction of dimensionality, decreasing the number of effective DOFs to be controlled [9]. Specifically, they observed that most ($> 80\%$) of the joint kinematic variance during grasping could be described by only two synergies. Subsequent studies confirmed similar results [10], [11], [12], [13], [14].

From a mathematical point of view, these researchers approached the study of variance by means of matrix decomposition techniques. Specifically, they took advantage of Singular Value Decomposition (SVD) [15] and Principal Component Analysis (PCA) [16]. These methods allow the decomposition of a non-square matrix - containing the collected kinematic data of the hand - into singular vectors (or principal components) and associated singular values (or principal component scores) to investigate the distribution of variance. However, pre-processing of the hand kinematic data before performing the decomposition plays a crucial role. For example, [9] centered each joint's data around its mean value, [12] considered two cases: joint data re-scaled to contain values between 0 and 1, and joint data re-scaled to have unit variance, while [14] used the raw data binned to fixed increments of time between start and end of a trial. These pre-processing approaches led to similar conclusions in terms of dimensionality reduction, but different quantitative estimates of the effective amount of this reduction. While some have explored how data structure affects synergy extraction [17], [18] and others have investigated how electromyography pre-processing changes the resultant muscle synergies [19], to our knowledge no-one has explored how pre-processing affects kinematic hand synergies.

In this work, we present an analysis of the effect of the most common pre-processing techniques on the dimensionality reduction problem. First, to provide intuitive understanding of the effect of the different pre-processing methods, we show a numerical example for a simulated 2 degree-of-freedom (DOF) system. Then to validate the intuition gained from the 2 DOF system, we apply the same approach to real hand kinematic data obtained from a subject performing a complex hand manipulation task: piano playing.

Studying piano playing has two intrinsic advantages. First, it is widely recognized to be a complex manipulation task [20] that requires specific training and years of practice [21]. Thus, it should exhibit a more refined distribution of variance than a simpler task such as object grasping. Second, playing piano requires limited range-of-motion trajectories of the joints, thanks to the constraint imposed by the piano keyboard. Joint trajectories are generally curved, but over small ranges of motion, a linear approximation may be satisfactory.

Funding was from a Ford Foundation Fellowship, a Takeda Fellowship and the Eric P. and Evelyn E. Newman Fund.

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2023 International Conference on Rehabilitation Robotics (ICORR) | 979-8-3503-4275-8/23/$31.00 ©2023 IEEE | DOI: 10.1109/ICORR58425.2023.10304710

Therefore, piano playing is an ideal task to evaluate linear mathematical tools such as Singular Value Decomposition. Moreover, the current literature [10], [11], [12], [13], [14] largely studies grasping of common household items, which approximates a lower bound on the repertoire of the hands' capabilities. However, if we want to restore activities of daily living that involve manipulation it is important to specifically study hand manipulation. Studying piano playing can help approximate an upper bound on the hands' capabilities.

The results presented in this study help to shed light on the quantification - and correct interpretation - of the synergistic behavior of the human hand. These results have implications for the design of assistive and rehabilitative technologies.

II. METHODS

A. Theoretical Background

Consider a data matrix $X \in \mathbb{R}^{n \times m}$, where n represents the number of observations (e.g., the time evolution of a joint angle or degree of freedom) and m represents the number of features (e.g., the number of analyzed DOFs) for a given task. The main goal in the study of kinematic hand synergies is to mathematically process i.e., decompose, the matrix X to identify patterns (synergies) that could be related to a reduction of the dimensionality of the control problem. This is a proposed solution to the infamous "curse of dimensionality," which states that higher degree-of-freedom systems are more difficult to control [22]. Dimensionality reduction is often found via SVD.

In SVD, the matrix X is decomposed into the product of an orthonormal matrix U, a matrix with values only on its main diagonal S, and another orthonormal matrix V:

$$X = U \cdot S \cdot V^T \tag{1}$$

where $U \in \mathbb{R}^{n \times n}$, $S \in \mathbb{R}^{n \times m}$ and $V \in \mathbb{R}^{m \times m}$. From a geometric perspective, these operations could be interpreted as the succession of a rotation U, a stretch S and another rotation V^T. Moreover, the principal diagonal elements of S, also known as singular values σ_i, are related to the eigenvalues λ_i of the square matrix $X^T \cdot X$ by the following relationship: $\sigma_i^2 = \lambda_i$ for $i = 1, ..., m$.

Mathematically, SVD provides information about the matrix X's distribution over the hyperspace $\mathbb{R}^{n \times m}$; of particular interest is the data distribution over the feature space $\mathbb{R}^m$. The matrix V^T can be used to obtain the principal directions in the feature space, while the singular values - obtained from S - return an estimate of the magnitude of variance projected onto each principal direction.

To obtain a measure of the data variance-accounted-for (VAF) by a given synergy, we used the singular values along the principal diagonal of matrix S. Specifically, we report VAF as a decimal, ranging from 0 to 1 obtained by dividing the square of a singular value by the total sum of the squares of the singular values: $VAF(i) = \sigma_i^2 / \sum \sigma_i^2$ for $i = 1, ..., m$.

Pre-processing operations performed on the data matrix, X, will, of course, affect the singular value decomposition and the resultant estimation of the related VAF for a given analyzed motion. The important question is how and how much.

In the work presented here, we considered four different pre-processing operations that have previously been used in the literature for kinematic hand synergy decomposition: (1) no pre-processing [14], (2) removing the mean [9], (3) z-score [12], and (4) range 0-1 [12]. Mathematically, the latter three approaches result in three new data matrices:

$$X_{rm} = X - mean(X) \tag{2}$$

$$X_{zs} = \frac{X - mean(X)}{std(X)} \tag{3}$$

$$X_{r01} = \frac{X - min(X)}{max(X - min(X))} \tag{4}$$

where X_{rm} represents a data matrix with the mean removed, X_{zs} represents a z-scored data matrix, and X_{r01} represents a data matrix scaled between 0 and 1. Excluding trivial cases in which the data matrix features $x(1:n, m)$ already have either zero mean, unit variance or a 0-1 range, it is evident that the singular values and related orthogonal directions $v_i \in V^T$ will change due to pre-processing.

B. Numerical Validation

A set of numerical simulations was performed in Matlab 2022b (Mathworks, USA) to observe the changes in singular value decomposition due to the different pre-processing methods. To provide an intuitive understanding, a 2 DOF system ($m = 2$) was considered to facilitate graphical interpretation of the numerical simulations. The two DOFs were linearly correlated with the addition of external noise:

$$x_2(i) = k_1 x_1(i) + k_0 + \epsilon(i) \quad i = 1, ..., N \tag{5}$$

where $x_1(t), x_2(t)$ represent two features of the system, k_1, k_0 are the linear correlation parameters between these features, $\epsilon(t)$ is an external disturbance, and N represents the number of simulated points. The results presented in Section III-A used the following simulation parameters: $k_1 = 0.50$, $k_0 = 3$, and $N = 101$. SVD was performed on the data matrix $X = [x_1, x_2] \in \mathbb{R}^{N \times 2}$ and the results of all the proposed pre-processing techniques.

C. Piano Playing

To validate the intuition provided by the numerical example, a preliminary experimental validation on real human hand kinematic data was performed with a single subject[1] He was informed about the experimental procedure and agreed to sign a consent form. All procedures were approved by MIT's Institutional Review Board. performing a complex manipulation task: playing the piano (Fig. 1). Piano playing was selected due to its evident complexity and high level of required skill. This was expected to address a common weakness of synergy analysis via SVD: most of the data

[1]The subject was a 31 year old, right-handed skilled piano playing male who had 10 years of trained piano practice and 20 years of overall piano experience.

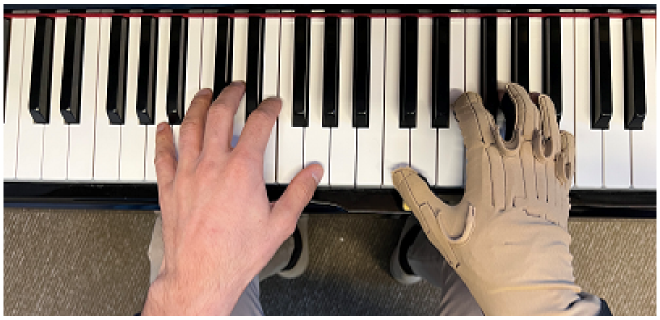

Fig. 1: **Experimental setup** of one subject playing the piano while kinematic data of their right hand was recorded with a CyberGlove.

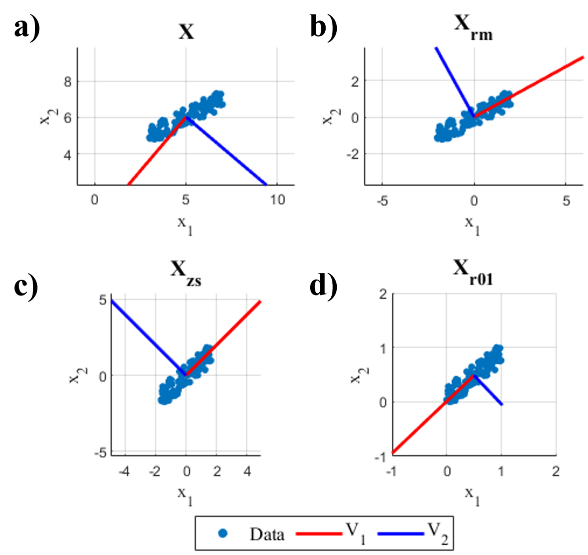

Fig. 2: **Numerical validation data** of the various pre-processing types are shown by the light blue dots. Additionally, the computed 1^{st} and 2^{nd} eigenvectors (i.e. synergies) are shown by the red and blue lines, respectively. (a) denotes the raw data while (b), (c) and (d) denote the data after the pre-processing described by Eq. 2, 3, and 4, respectively.

variance is 'lumped' into the first few synergies[2], compromising the reliability of numerical differences between the higher-order synergies. Piano-playing was expected to evoke a wider variability of the extracted synergies, thus avoiding the effect of cumulative lumping of VAF into the first few synergies. This would provide a better understanding of the effect of different pre-processing methods.

The subject separately performed a set of 7 piano pieces (Chopin: Nocturne Op. 9 No. 2; Chopin: Waltz Op. 64 No.1; Giovanni Allevi: Come Sei Veramente; Hans Zimmer: Oogway Ascends; W.A. Mozart: Turkish March; Christopher Norton: A Whimsy; Paul Desmond: Take Five) while wearing a CyberGlove (CyberGlove; Virtual Technologies, Palo Alto, CA), a glove with embedded sensors that measure joint kinematics (Fig. 1). Specifically, the flexion of the distal interphalangeal (DIP), proximal interphalangeal (PIP), and metacarpophalangeal (MCP) joints of the four fingers were measured. Additionally, the abduction (ABD) of the four fingers at the MCP joints was measured. At the thumb, the flexion at the MCP and interphalangeal (IP) joints, abduction (ABD) at the carpometacarpal joint, and rotation (ROT) about an axis passing through the trapeziometacarpal joint were measured. Lastly, palm arch (PA) and wrist (W) pitch and yaw were measured. The subject wore the CyberGlove on his right hand. The CyberGlove collected samples at ~ 200 Hz with a nominal angular resolution of $< 0.1°$.

Synergies (i.e., linear combinations of the joint DOFs) and their VAF were extracted using the SVD algorithm presented above. To identify the number of significant synergies, we followed the method of [18]. Specifically, we report the number of synergies required to achieve at least 90% VAF and where inclusion of another subsequent synergy did not add an additional 5% VAF. To determine if there was an effect of pre-processing on the number of significant synergies, these values were submitted to a 4 (pre-processing type) x 1 repeated-measures ANOVA.

Moreover, we compared the calculated kinematic hand synergies, V, across pre-processing types. To do so, we computed the product of each individual piece's synergies identified by a pre-processing type with those of a different pre-processing type:

$$C = |cos(\theta)| = |V^T_{preprocessing1} \cdot V_{preprocessing2}| \quad (6)$$

This resulted in a matrix consisting of the cosine similarities, $C(i, j)$, between synergies where i denotes the i^{th} synergy of pre-processing 1 and j denotes the j^{th} synergy of pre-processing 2. Here, we report the magnitude of these cosine similarities, ranging from 0 to 1. If synergies were the same irrespective of pre-processing (i.e. the data spans the same hyperspace), we would expect a matrix of cosine similarities with ones on the diagonal and zeros otherwise. In all other cases (i.e. $V_{preprocessing1} \neq V_{preprocessing2}$), the resulting matrix, $C \in \mathbb{R}^{m \times m}$, will be an asymmetric square matrix.

All data processing and statistical analyses were performed using custom scripts in MATLAB. The significance level for statistical tests was $\alpha = 0.05$.

III. RESULTS

A. Numerical Validation

In Fig. 2 we present singular vector decomposition of numerical data as described in Section II-B. There the raw and pre-processed data can be seen. It is seen that in the raw data case, the first eigenvector does not appropriately represent the slope of the data (Fig. 2a).

The coefficients of the first and second eigenvectors are shown in Fig 3a-b. Given that the slope of the data was 0.50 (i.e., $k_1 = 0.50$), we expect the ratio of the coefficients of the first eigenvector to equal ~ 0.50 with some error due to the added noise (i.e., $V(2,1)/V(1,1) \sim 0.50$). The observed

[2]Note, in the piano playing study the angles of the 23 joints of the hand comprise the feature space. Thus, a reported synergy is a vector (hyperspace) representing a common hand configuration which is described by angles of the joints in the hand.

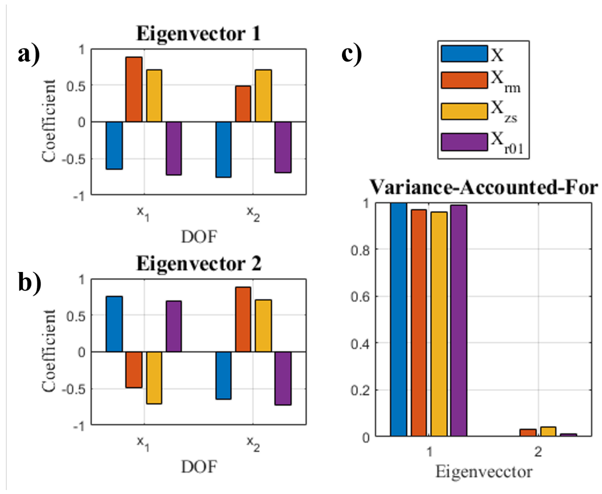

Fig. 3: **Eigenvectors and VAF of the numerical validation data.** Different colors denote different pre-processing types.

	X	X_{rm}	X_{zs}	X_{r01}
Avg	2.29	6.14	9.00	1.14
SD	0.49	0.90	1.63	0.38

TABLE I: **Number of significant synergies** in the piano playing study based on data pre-processing.

ratio was 1.177, 0.550, 1.000, and 0.952 in the raw, mean removed, z-score, and range 0-1 data, respectively. Note, the mean values of the raw data are 6.022 and 5.013; their ratio is 1.201. Additionally, the mean values of the range 0-1 data are 0.486 and 0.503; their ratio is 0.967. This indicates when the data is not centered, the first synergy is directed towards the mean of the data.

Fig 3c shows the VAF of each eigenvector. The VAF of the first eigenvector was 0.995, 0.968, 0.957, and 0.989 in the raw, mean removed, z-score, and range 0-1 data, respectively. Evidently, when the data was not pre-processed, the second synergy was considered to negligibly contribute to the variation of the data. While it may appear to be an advantage to have a system adequately described by fewer synergies, it is important to recall that in the raw data case the first synergy was incorrect; rather than representing the co-variation in the data it reflected the data mean.

B. Piano Playing

Fig. 4 demonstrates the VAF in the piano experiments averaged across pieces for each pre-processing type. It is seen that, on average, two synergies achieve 0.90 VAF when data is not pre-processed or set to range from 0 to 1. However when the data is removed of the mean or z-scored, 6 and 9 synergies (respectively) are needed to reach 0.90 VAF.

We quantified the number of significant synergies based on each data pre-processing type. The average and standard deviation is reported in Table I. A one-way ANOVA revealed a significant effect of pre-processing on the number of significant synergies ($F_{3,24} = 94.47, p = 1.99e-13$). Post-hoc t-tests (Bonferroni corrected $\alpha = 0.05/6 = 0.0083$) revealed that the range 0-1 data had a statistically fewer

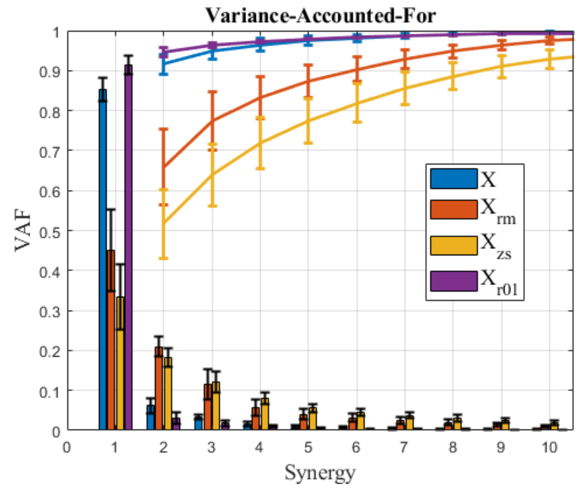

Fig. 4: **VAF of each synergy in the piano experiment** averaged across piece. Different colors denote different pre-processing types. Lines show the cummulative sum of the VAF. Errorbars are $\pm$1SD. Due to space constraints, only the first 10 synergies are shown.

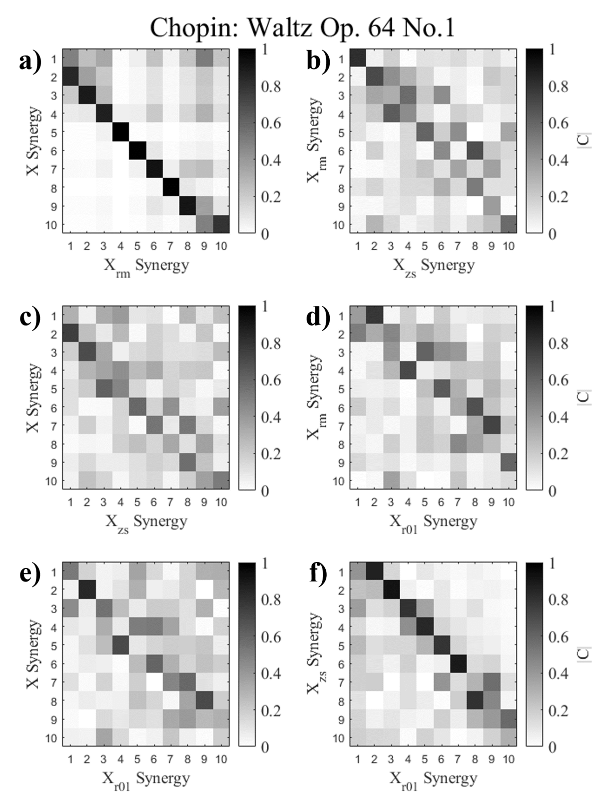

Fig. 5: **Comparison of synergies across different pre-processing types** for one representative piece. The matrix of cosine similarities between synergies of different pre-processing types is presented for the Chopin Waltz Op. 64 No. 1. A black value denotes a cosine similarity of 1 and a white value denotes a cosine similarity of 0. Due to space constraints, comparisons of only the first 10 synergies, ordered by decreasing VAF, are shown.

number of significant synergies than the raw data ($p = 3.67e-04$), data with the mean removed ($p = 1.23e-08$), and z-score data ($p = 3.35e-08$). Moreover, the raw data had a statistically fewer number of significant synergies than the data with the mean removed ($p = 3.70e-10$), and z-score data ($p = 2.28e-07$). Further, the data with the mean removed had a statistically fewer number of significant synergies than the z-score data ($p = 0.0016$).

To compare the synergies across pre-processing type, we computed the magnitude of the similarity coefficients between the matrices of synergies. The 6 comparisons, stemming from the pairwise combinations of the 4 pre-processing types (i.e., $\binom{4}{2} = 6$), are reported in Fig. 5 for a representative piece. If two types of pre-processing methods lead to similar synergies we would observe a matrix of ones on the diagonal and zeros otherwise. Clearly this is not the case. In fact, when comparing the raw data synergies with the ones obtained from removing the mean (Fig. 5a), we observe that there is a large similarity in the subdiagonal of the matrix of cosine similarities. High similarity is also seen in the superdiagonal when we compare the z-score data synergies and the range 0-1 synergies (Fig. 5f). These relations were also observed in the pieces not reported here.

IV. DISCUSSION

A. Effects of Centering the Data

The mean removed and z-scored data are centered while the raw and the range 0-1 data are not. In Fig 3c, it is seen that the first synergy of non-centered data has a greater VAF than the centered data. This was also observed in the piano study's resultant synergies (Fig. 4). Due to the large first synergy VAF, the non-centered data had statistically fewer significant synergies than the centered data (Table I).

In the validation data, Fig 2a showed that the first eigenvector did not align well with the expected principal direction of the data. Recall that the validation data was set up using Eq. 5, where the slope, k_1, was 0.50. Thus, we expected the ratio of the coefficients of the first eigenvector (i.e., $V(2,1)/V(1,1)$) to be 0.50. This was not the case for the raw data – $V(2,1)/V(1,1) = 1.18$. Fig 2a suggests that this first synergy was used to reach the center of the data, resulting in an eigenvector whose coefficients did not tell us how our data co-varied. Moreover, this relationship was also observed in the piano data. The cosine similarity matrix was computed to compare synergies across pre-processing types (Fig. 5). If the synergies were similar, we would have observed a matrix of ones on the diagonal and zeros otherwise. Figure 5a shows a matrix that contains high similarity on the subdiagonal when comparing the raw data synergies to the ones obtained from removing the mean. Given this in conjunction with the observation made in Fig 2a, we hypothesize that the first synergy in piano playing is similarly used to center the data, while the subsequent synergies inform us how the joints of the hand co-vary. Moreover in Fig. 5e, the cosine similarity matrix between the z-score data synergies and the range 0-1 data synergies has high similarity in the superdiagonal. Again, the range 0-1 data is not centered, suggesting that the first synergy centers the data, while the subsequent synergies inform us how the joints of the hand co-vary. In sum, not centering the data leads to a first-synergy dominated behavior that is not representative of the principal directions of motion.

B. Effects of Changing Data Variance

The z-scored and range 0-1 data change the variance of each DOF while the raw data and the data with the mean removed do not. Specifically, z-scoring forces each DOF to unit variance. As such, we are unable to determine how much individual DOFs co-vary with one another. This is represented by the eigenvectors of the data presented in Sections II-B and III-A. We set up these data using Eq. 5, where the slope, k_1, was 0.50. Thus, we expected the ratio of the coefficients of the first eigenvector (i.e., $V(2,1)/V(1,1)$) to be 0.50. In the z-scored and range 0-1 data this ratio was 1.00 and 0.95, respectively. Because the variance of the data was changed, information about how joint motions changed in relation to one another was lost. Thus, the ratio of the first eigenvector coefficients was constrained to 1.00, due to z-scoring, as opposed to the expected value of 0.50.

Applying this understanding to kinematic hand synergies, we conclude that reducing the variance of the data would lead us to a synergy that does not describe how one joint varies with another, rather just that the two are related. For example, let's say x_1 and x_2 in Eq. 5 represent flexion/extension of the index PIP and DIP. Z-scoring the data would result in a synergy that whenever the PIP flexes (or extends) the DIP also flexes (or extends) the same amount. However, based on the data, this is not true; whenever the PIP flexes (or extends), the DIP flexes (or extends) half that amount. To conclude, changing the variance of the data during pre-processing would result in a synergy that does not describe how the several DOFs co-vary.

In this study, we do not have a physiological ground truth of the synergies used in the piano playing task as we do not have access to humans' internal model. However, we can derive a reasonable understanding of the dimensionality reduction provided by the linear combinations of hand movements using our knowledge of linear algebra and the intuition provided by the numerical example in Section III-A. In sum, not centering the data during pre-processing will result in a dominant first synergy that aims to center the data (Fig. 2a and 2d), leading to a greatly reduced significant number of synergies. Moreover, changing the variance of the data during pre-processing will lead to a synergy that demonstrates that certain DOFs do, indeed, co-vary but it will not quantify how much (i.e., the ratio between them). Thus, if you are interested in conducting SVD to both accurately estimate the number of significant synergies and how each DOF co-varies with one another in a given synergy, you should center your data but not change its variance. This is consistent with the pre-processing step of removing the mean.

C. A Geometrical Understanding

Understanding that Singular Value Decomposition decomposes a data matrix, X, into a product of a rotation matrix,

U, a (diagonal) stretch matrix, S and another rotation matrix, V^T provides insight into how pre-processing methods may affect the decomposed synergies. Specifically, a pure rotation of the data matrix, X, would lead to a pure rotation of the eigenvectors. Moreover, a heterogeneous stretch of each DOF in X will lead to heterogeneous changes of the eigenvalues, resulting in different VAFs. In the z-score X_{zs} and range 0-1 X_{r01}, transformations presented here, each feature (i.e., joint DOF) is separately scaled; that is a heterogeneous stretch. Thus, the VAF of each feature will be accordingly altered.

D. Implications for Robotic Rehabilitation

The design of rehabilitative devices for the hand is still in search of the best trade-off between number of actuated DOFs, and device function, appearance, and comfort (e.g., minimized weight and size). In this scenario, prosthetic hand designs that exploit fewer actuators to move multiple joints represent a growing portion of the research prototypes [23], [24], [25], [26]. Moreover, clinicians have used synergies as a basis for rehabilitation post Cerebral Vascular Accident [27], [28]. Thus, the understanding of how pre-processing affects synergy decomposition presented here can better inform the selection of the quantity and kinematic coupling of synergies that assistive and prosthetic hand devices should implement.

V. CONCLUSION

This work investigated the role of data pre-processing on the extraction of kinematic hand synergies. Using numerical simulation and human hand kinematic data of a subject performing playing the piano, we showed that removing the mean appears to be the best approach to minimize error in the interpretation of computed synergies and their related VAF. This understanding may inform the design of devices that replicate and rehabilitate the human hand. Future work will aim to expand the study of shoulder, arm and hand kinematics during piano playing to uncover the role of synergy decomposition in this complex manipulation task.

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